

Heat is a form of energy that is transferred from one body to another body at a lower temperature by virtue of the temperature difference between the bodies.

In general there are three ways by which heat may be transferred: ~~given~~ By conduction, By convection and By radiation.

### By Conduction

Heat conduction is the transfer of heat from one part of a substance to another, without appreciable displacement of the molecules forming the substance. Heat would be transferred by conduction through a metal if one end is higher in temperature than the other.

### By Convection

Heat transfer by convection is the transfer of heat within a fluid by mixing of one portion of the fluid with another. The movement of the fluid may be caused by differences in density resulting from the temperature differences as is natural convection (or free convection). For example the heat transferred from a hot plate to the atmosphere is by natural convection, whereas the

heat transferred by a domestic fan-heater, in which a fan blows air across an electric element, is by forced convection.

The heat transferred through solid bodies is by conduction only, whereas the heat transferred from a solid surface to a liquid or gas takes place partly by conduction and partly by convection.

### By Radiation

Above absolute zero, all matter continuously emits electromagnetic radiation. The higher the temperature, the greater the amount of energy radiated. If, therefore, two bodies at different temperatures are so placed that the radiation from each ~~other~~ body is intercepted by the other, then the body at the lower temperature will receive more energy than it is radiating, and hence, its internal energy will increase.

Similarly the internal energy of the body at the higher temperature will decrease. Thus there is a net transfer of energy from the higher



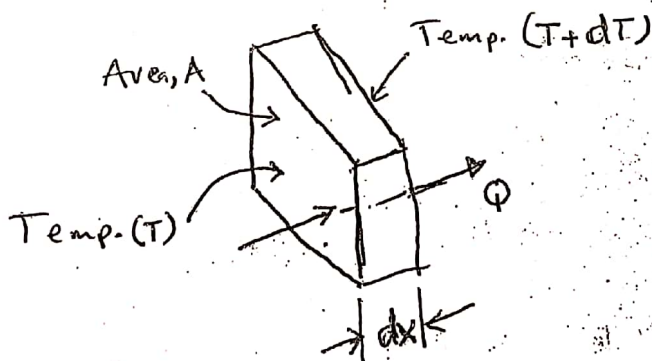
(3)

temperature body to the low temperature body by virtue of temperature difference between the bodies.

Radiant energy, being electromagnetic radiation requires no medium for its propagation and will pass through a vacuum.

### Fourier's Law of Conduction

Fourier's law states that the rate at which heat is conducted through a single homogeneous solid is directly proportional to the area of the section perpendicular to the direction of heat flow, and to the temperature difference with respect to the length of the path of the heat flow,  $dT/dx$ . This is illustrated in the figure below



A thin slab of thickness  $dx$  and surface area  $A$  has one face at ~~area~~ temperature  $T$  and the other at temperature  $(T + dT)$ . Applying Fourier's law for the rate of heat





$$\text{or } \dot{Q}_x = -A \int_{T_1}^{T_2} k dT$$

For most materials the thermal conductivity is approximately constant over a wide range of temperatures,

$$\text{i.e. } \dot{Q}_x = -AK \int_{T_1}^{T_2} dT$$

$$\text{or } Q = -\frac{KA}{x} (T_2 - T_1) = \frac{KA}{x} (T_1 - T_2) \dots \dots \dots (2)$$

Note that in this case the area in the direction at right angles to the heat flow remains constant through the slab.

Example 1

The inner surface of a plane brick is at 40°C and the outer surface is at 20°C. Calculate the rate of heat transfer per unit area of each surface; the wall is 250 mm thick and the thermal conductivity of the brick is 0.52 W/mK

Soln

$$\dot{Q} = \frac{KA}{x} (T_1 - T_2)$$

Therefore,  $\frac{\dot{Q}}{A} = \dot{q} = \frac{k}{x} (T_2 - T_1) = \frac{0.52}{250 \times 10^{-3}} (40 - 20)$

$$= 41.6 \text{ W/m}^2$$

where  $\dot{q}$  = rate of heat transfer per unit area.

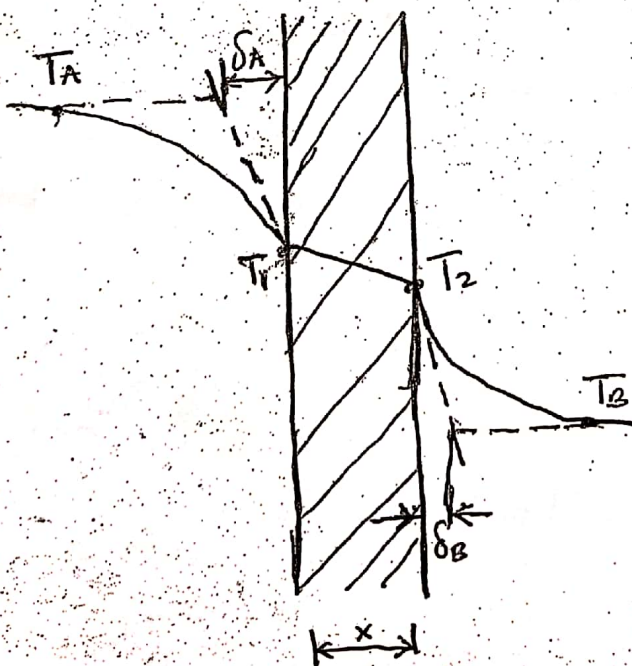
## Newton's Law of Cooling

Newton's law of cooling states that the heat transfer to or from a solid surface of area  $A$ , at a temperature  $T_w$ , to or from a fluid of temperature  $T$ , is given by

$$\dot{Q} = hA(T_w - T) \dots \dots \dots (3)$$

where,  $h$  is the heat transfer coefficient.  
The units of  $h$  are seen to be  $W/m^2 K$ , or  $kW/m^2 K$ .  
The heat transfer coefficient,  $h$ , depends on the properties of the fluid and on the fluid velocity.

The figure below illustrates the transfer of heat from a fluid A to a fluid B through a dividing wall of thickness  $x$ , and thermal conductivity  $k$ .





The temperature varies in the direction of heat transfer as shown. In fluid A the temperature decreases rapidly from  $T_A$  to  $T_1$  in the region of the wall, similarly in fluid B the temperature decreases rapidly from  $T_2$  to  $T_B$  in region of the wall. In most practical cases the fluid temperature is approximately constant throughout its bulk, apart from a thin layer near the solid surface bounding the fluid. The dotted lines drawn in the figure above show that the thickness of this film or layer of fluid is given by  $\delta_A$  for fluid A and  $\delta_B$  for fluid B. The heat transfer in this films is by conduction only, hence applying equation (2), considering a unit surface area, from fluid A to the wall

$$\dot{q} = \frac{k_A}{\delta_A} (T_A - T_1) \quad \dots \dots \dots (a)$$

from the wall to fluid B (b)

$$\dot{q} = \frac{k_B}{\delta_B} (T_2 - T_B)$$

Also from equation (3), from fluid A to the wall (c)

$$\dot{q} = h_A (T_A - T_1)$$

from the wall to fluid B (d)

$$\dot{q} = h_B (T_2 - T_B)$$

Equating equations (a) and (c), and equations (b) and (d), it is seen that  $h_A = \frac{k_A}{\delta_A}$  and  $h_B = \frac{k_B}{\delta_B}$

The heat flow through the wall in the figure is given by equation (16.2). For a unit surface area

$$\dot{q} = \frac{k}{x} (T_1 - T_2)$$

For steady flow state heat transfer, the heat flowing from fluid A to the wall is equal to the heat flowing through the wall, which is also equal to the heat flowing from the wall to fluid B. If this were not so, then the temperatures  $T_A$ ,  $T_1$ ,  $T_2$ , and  $T_B$  would not remain constant but would change with time. Therefore,

$$\dot{q} = h_A (T_A - T_1) = \frac{k}{x} (T_1 - T_2) = h_B (T_2 - T_B)$$

Rewriting the equations,

$$(T_A - T_1) = \frac{\dot{q}}{h_A}; \quad (T_1 - T_2) = \frac{\dot{q}x}{k}; \quad (T_2 - T_B) = \frac{\dot{q}}{h_B}$$

Adding corresponding sides of the three equations gives

$$(T_A - T_1) + (T_1 - T_2) + (T_2 - T_B) = \frac{\dot{q}}{h_A} + \frac{\dot{q}x}{k} + \frac{\dot{q}}{h_B}$$

Therefore,

$$(T_A - T_B) = \dot{q} \left( \frac{1}{h_A} + \frac{x}{k} + \frac{1}{h_B} \right)$$

$$\text{i.e. } \dot{q} = \frac{T_A - T_B}{\frac{1}{h_A} + \frac{x}{k} + \frac{1}{h_B}}$$

By analogy with equation (3)

$$\dot{q} = U_B (T_A - T_B) \quad \dots \dots \dots (4)$$

$$\text{or } \dot{q} = U_A (T_A - T_B) \quad \dots \dots \dots (5)$$



where,  $\frac{1}{U} = \left( \frac{1}{h_A} + \frac{x}{k} + \frac{1}{h_B} \right)$  . . . . . (6)

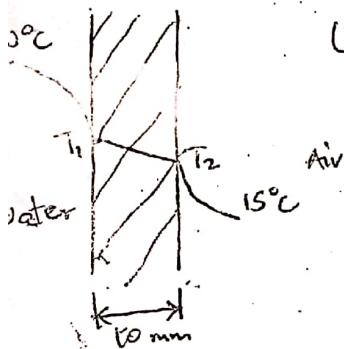
$U$  is called the overall heat transfer coefficient and it has the same units as  $k$ .

Example 2

A mild steel tank of wall thickness 10 mm contains water at  $90^\circ\text{C}$  when the atmospheric temperature is  $15^\circ\text{C}$ . The thermal conductivity of mild steel is  $50 \text{ W/mK}$ , and the heat transfer coefficients for the inside and outside of the tank are 2800 and  $11 \text{ W/m}^2\text{K}$  respectively. Calculate:

- i. the rate of heat loss per unit area of tank surface
- ii. the temperature of the outside surface of the tank.

Soln



(i) From equation (6) and the diagram at the side,

$$\begin{aligned} \frac{1}{U} &= \frac{1}{h_A} + \frac{x}{k} + \frac{1}{h_B} = \frac{1}{2800} + \frac{10}{50 \times 10^3} + \frac{1}{11} \\ &= 0.000357 + 0.0002 + 0.0909 \\ &= 0.0915 \end{aligned}$$

From equation (4),

$$\begin{aligned} \dot{q} &= U(T_A - T_B) \\ &= \frac{90 - 15}{0.0915} = 820 \text{ W/m}^2 \end{aligned}$$

(ii) From equation (3)

$$\dot{q} = h_B(T_2 - T_B) \text{ or } 820 = 11 \times (T_2 - 15)$$

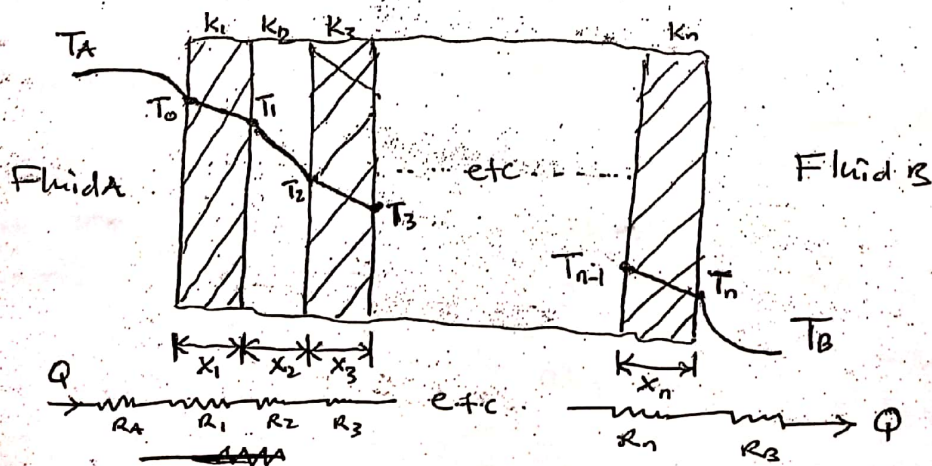
$$T_2 = \frac{820}{11} + 15 = 89.6^\circ\text{C}$$

$\therefore$  Temperature of outside surface of tank =  $89.6^\circ\text{C}$ .

## The Composite Wall and the Electrical Analogy

There are many cases in practice when different materials are constructed in layers to form a composite wall. An example of this is the wall of a building, which usually consists of a layer of plaster, a row of bricks, an air gap, a second row of bricks, and perhaps a cement rendering on the outside surface.

The figure below shows a composite wall with  $n$  layers of material with thickness  $x_1, x_2, x_3, \dots$  and of thermal conductivity  $k_1, k_2, k_3, \dots$ . On one side of the composite wall there is a fluid A at temperature  $T_A$ , and the heat transfer coefficient from fluid to wall is  $h_A$ ; on the other side of the composite wall there is a fluid B, and the heat transfer coefficient from wall to fluid is  $h_B$ . Let the temperature of the wall in contact with fluid A be  $T_0$  and the temperature of the wall in contact with fluid B be  $T_n$ ; the intermediate temperatures are then  $T_1, T_2, T_3, \dots$  as shown.





$$\frac{KJ}{s} = kW$$

This problem can be solved by using an electrical analogy. The flow of heat can be thought of as analogous to an electric current. The heat flow is caused by a temperature difference whereas the current flow is caused by a potential difference, hence it is possible to postulate a thermal resistance analogous to an electrical resistance. From Ohm's law

$$V = IR \quad \text{or} \quad I = V/R$$

where,  $V$  is potential difference,  $I$  is current and  $R$  is resistance.

Comparing this equation with equation (2),  $\dot{Q} = (KA/x)(T_1 - T_2)$

Thermal resistance,  $R = \frac{x}{KA}$  ----- (7)

where,  $\dot{Q}$  is analogous to  $I$ , and  $(T_1 - T_2)$  is analogous to  $V$ .

The composite wall is analogous to a series of resistors as shown in the figure above, and resistances in series can be added to give the total resistance. To find the resistance of a fluid film, it is necessary to compare Ohm's law with equation (3),  $\dot{Q} = hA(T_w - T)$

i.e. Thermal resistance of a fluid film,  $R = \frac{1}{hA}$  ----- (8)

where  $\dot{Q}$  is analogous to  $I$  and  $(T_w - T)$  is analogous to  $V$ . The units of thermal resistance are  $K/W$  or  $K/KW$

Referring to the figure above,

$$R_A = \frac{1}{h_A A}, \quad R_1 = \frac{x_1}{k_1 A}, \quad R_2 = \frac{x_2}{k_2 A}, \quad \text{etc.}, \quad R_n = \frac{x_n}{k_n A} \quad \text{and} \quad R_B = \frac{1}{h_B A}$$

The total resistance to heat flow is

$$R_T = R_A + R_1 + R_2 + R_3 + \dots + R_n + R_B = \frac{1}{h_A A} + \frac{x_1}{k_1 A} + \text{etc} + \frac{x_n}{k_n A} + \frac{1}{h_B A}$$

Or for any number of layers of material

$$\text{Total resistance, } R_T = \frac{1}{h_A A} + \sum \frac{x}{kA} + \frac{1}{h_B A}$$

It is seen from equation (9), that in this case, the surface area,  $A$ , remains constant through the wall, and it is usual to calculate the total resistance for unit surface area in such problems.

Using the electrical analogy for the overall heat transfer,

$$\dot{Q} = \frac{T_A - T_B}{R_T} \quad (10)$$

In equation (8) the overall heat transfer coefficient,  $U$ , is given as

$$\frac{1}{U} = \frac{1}{h_A} + \frac{x}{k} + \frac{1}{h_B}$$

For any number of walls,

$$\frac{1}{U} = \frac{1}{h_A} + \sum \frac{x}{k} + \frac{1}{h_B}$$

It is seen that the reciprocal of  $U$  is simply the thermal resistance for unit area,

$$\text{i.e. } \frac{1}{U} = R_T A \quad \text{or} \quad U = \frac{1}{R_T A} \quad (11)$$

If the inner and outer surface temperatures are known then the heat transfer can be found by calculating the thermal resistance of the composite wall only,

$$\text{i.e. } R = \sum \frac{x}{kA}$$

The overall heat transfer coefficient from one wall to the other is given by

$$\frac{1}{U} = \sum \frac{x}{k}$$



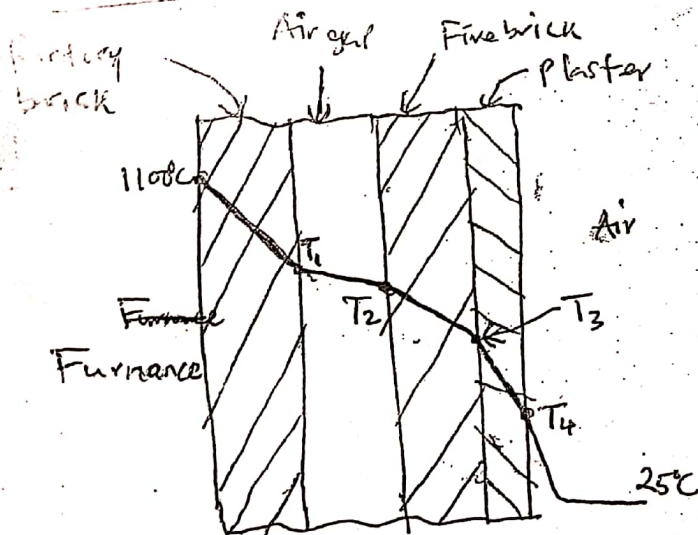
Example 3

A furnace wall consists of 25 mm wide refractory brick and 125 mm wide insulating firebrick separated by an air gap. The outside wall is covered with 12 mm thickness of plaster. The inner surface of the wall is at  $1100^{\circ}\text{C}$  and the room temperature is  $25^{\circ}\text{C}$ . The heat transfer coefficient from the <sup>outside</sup> outside wall surface to the air in the room is  $17 \text{ W/m}^2\text{K}$ , and the resistance to heat flow of the air gap is  $0.16 \text{ K/W}$ . The thermal conductivities of refractory brick, insulating brick and plaster are 1.6, 0.3 and  $0.14 \text{ K/W}$   ~~$\text{K/W}$~~   $\text{W/mK}$ , respectively.

Calculate:

- i) the rate of heat loss per unit area of wall surface
- ii) the temperature at each interface throughout the wall
- iii) the temperature at the outside surface of the wall

Soln



~~Resistance of refractory brick =~~

$$\text{Resistance of refractory brick} = \frac{25}{10^3 \times 1.6} = 0.0781 \text{ K/W}$$

$$\text{Resistance of insulating fire brick} = \frac{125}{10^3 \times 0.3} = 0.417 \text{ K/W}$$

$$\text{Resistance of plaster} = \frac{12}{10^3 \times 0.14} = 0.0857 \text{ K/W}$$

Also using equation (8) for a fluid film,  $R = 1/hA$

$$\text{Resistance of air film on outside surface} = \frac{1}{17} \text{ K/W}$$

Hence,

$$\text{Total resistance, } R_T = 0.0781 + 0.417 + 0.0857 + \frac{1}{17} + 0.16$$

where the resistance of the air gap is 0.16 K/W

$$\text{i.e. } R_T = 0.8 \text{ K/W}$$

From equation (10)

$$\dot{Q} = \frac{T_A - T_B}{R_T} = \frac{1100 - 25}{0.8} = 1344 \text{ W}$$

i.e. Rate of heat loss per square metre of surface area = 1344 K/W

(ii) Referring to the above diagram figure, the interface temperatures are  $T_1$ ,  $T_2$ , and  $T_3$ ; the outside surface is at  $T_4$ . Applying electrical analogy to each layer and using the values of thermal resistance calculated above,

$$\dot{Q} = 1344 = \frac{1100 - T_1}{0.0781} \text{ i.e. } T_1 = 1100 - (1344 \times 0.0781) = 995^\circ$$

$$\text{Also } \dot{Q} = 1344 = \frac{T_1 - T_2}{0.16} \text{ i.e. } T_2 = 995 - (0.16 \times 1344) = 780^\circ \text{C}$$

$$\dot{Q} = 1344 = \frac{T_2 - T_3}{0.417} \text{ i.e. } T_3 = 780 - (1344 \times 0.417) = 220^\circ \text{C}$$



And  $\dot{\phi} = 1344 = \frac{T_3 - T_4}{0.0857}$ , i.e.  $T_4 = 220 - (13.44 \times 0.0857) = 104^\circ\text{C}$

1) The temperature  $T_4$  can also be found by considering the air film,

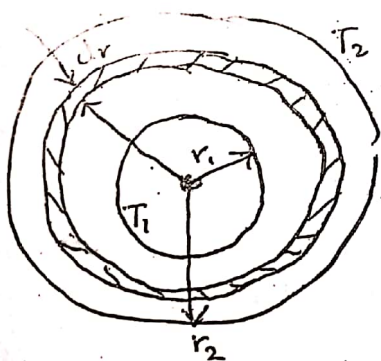
i.e.  $\dot{\phi} = 1344 = \frac{T_4 - 25}{1/17}$ , i.e.  $T_4 = (1344 \times \frac{1}{17}) + 25 = 104.1^\circ\text{C}$

i.e. Temperature at outside surface of wall =  $104.1^\circ\text{C}$

Heat Flow Through a Cylinder and Sphere

The Cylinder

Consider a cylinder of internal radius  $r_1$  and external radius  $r_2$  as shown in figure below. Let the inside and outside surface temperatures be  $T_1$  and  $T_2$ , respectively. Consider the heat flow through a small element of thickness  $dr$ , at any radius  $r$ , where the temperature is  $T$ . Let the thermal conductivity of the material be  $k$ . Then applying equation (1) for a unit length  $L$  of the cylinder



$$\dot{\phi} = -kA \frac{dT}{dx} = -k(2\pi r L) \frac{dT}{dr}$$

i.e.  $\dot{\phi} \frac{dr}{r} = -2\pi k L dT$

Integrating between inside and outside surfaces

$$\dot{\phi} \int_{r_1}^{r_2} \frac{dr}{r} = -2\pi k L \int_{T_1}^{T_2} dT$$

where  $\dot{\phi}$  and  $k$  are both constant

$$\dot{\phi} \log_e \frac{r_2}{r_1} = -2\pi k L (T_2 - T_1) = 2\pi k L (T_1 - T_2)$$

i.e.  $\dot{\phi} = \frac{2\pi k L (T_1 - T_2)}{\log_e (r_2/r_1)}$  --- (12)

Now from equation (2)

$$\dot{Q} = \frac{kA}{x} (T_1 - T_2)$$

Substituting a mean Area  $A_m$  into this equation and substituting also for the thickness  $x = (r_2 - r_1)$

$$\dot{Q} = \frac{kA_m (T_1 - T_2)}{r_2 - r_1}$$

Comparing above this equation with equation (12), then

$$\dot{Q} = \frac{kA_m (T_1 - T_2)}{r_2 - r_1} = \frac{2\pi kL (T_1 - T_2)}{\log_e (r_2/r_1)}$$

Therefore,

$$\frac{A_m}{r_2 - r_1} = \frac{2\pi L}{\log_e (r_2/r_1)}$$

$$A_m = \frac{2\pi L (r_2 - r_1)}{\log_e (r_2/r_1)} = \frac{A_2 - A_1}{\log_e (r_2/r_1)}$$

Here  $A_m$  is called the logarithmic mean area. It is seen from the above that there is also a logarithmic mean radius given by

$$r_m = \frac{r_2 - r_1}{\log_e (r_2/r_1)}$$

In the case of composite cylinders (e.g. a metal pipe with several layers of lagging) the most convenient approach is again that of electrical analogy;

$$R = \frac{x}{kA_m}$$

where  $x$  is the thickness of a layer and  $A_m$  is the logarithmic mean area for that layer.



From equation (12), applying the electrical analogy ( $I = V/R$ ),  
it is seen that (17)

$$R = \frac{\log_e(r_2/r_1)}{2\pi kL}$$

The film of fluid on the inside and outside surfaces can be treated as before using equation (8),

$$\text{i.e. } R_{\text{outside}} = \frac{1}{h_o A_o}$$

where  $A_o$  is the outside surface area,  $2\pi r_2 L$ , referring to the figure above and  $h_o$  is the heat transfer coefficient for the outside surface.

$$R_{\text{inside}} = \frac{1}{h_i A_i}$$

where  $A_i$  is the inside surface area,  $2\pi r_1 L$  and  $h_i$  is the heat transfer coefficient for the inside surface.

It is seen from equation (12)

$$\dot{Q} = \frac{2\pi kL (T_1 - T_2)}{\log_e(r_2/r_1)}$$

that the heat transfer rate depends on the ratio of the radii,  $r_2/r_1$ , and not on the difference  $(r_2 - r_1)$ . The smaller the ratio,  $r_2/r_1$ , then the higher is the heat flow for the same temperature difference.

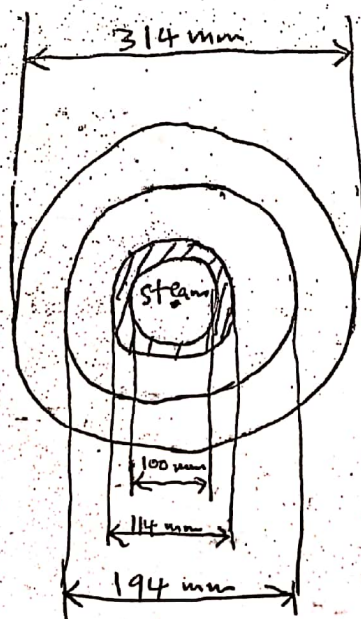
### Example 4

A steel pipe of 100 mm bore and 7 mm wall thickness carrying steam at  $260^{\circ}\text{C}$ , is insulated with 40 mm of a moulded high-temperature diatomaceous earth covering this covering in turn insulated with 60 mm of asbestos felt. The atmospheric temperature is  $15^{\circ}\text{C}$ . The heat transfer coefficient for the inside and outside surfaces are 550 and  $15 \text{ W/m}^2\text{K}$  respectively, and the thermal conductivities of steel, diatomaceous earth, and asbestos felt are 50, 0.09 and  $0.07 \text{ W/mK}$  respectively. Calculate:

- i) the rate of heat loss by steam per unit length of pipe
- ii) the temperature of the outside surface

### Soln

A cross-section of the pipe is shown below. Consider 1 m length of the pipe.





$$R = \frac{1}{hA} \quad \text{from equation (8)}$$

$$\text{Resistance of steam film} = \frac{10^3}{550 \times 2\pi \times 50 \times 1} = 0.00579 \text{ K/W}$$

From equation (13)

$$R = \frac{\log_e(r_2/r_1)}{2\pi kL}$$

Therefore,

$$\text{Resistance of pipe} = \frac{\log_e(57/50)}{2\pi \times 50 \times 1} = 0.000417 \text{ K/W}$$

Similarly

$$\text{Resistance of diatomaceous earth} = \frac{\log_e(97/57)}{2\pi \times 0.09 \times 1} = 0.94 \text{ K/W}$$

$$\text{and Resistance of asbestos felt} = \frac{\log_e(157/97)}{2\pi \times 0.07 \times 1} = 1.095 \text{ K/W}$$

From equation (8), for the air film on the outside surface

$$\text{Resistance of air film} = \frac{1}{hA} = \frac{10^3}{15 \times 2\pi \times 157 \times 1} = 0.0675 \text{ K/W}$$

Here

$$\begin{aligned} \text{Total resistance } R_T &= 0.00579 + 0.000417 + 0.94 + 1.095 + 0.0675 \\ &= 2.1087 \text{ K/W} \end{aligned}$$

$$\dot{Q} = \frac{T_A - T_B}{R_T} = \frac{260 - 15}{2.1087} = 116 \text{ W}$$

i.e. Rate of heat loss per metre length of pipe = 116 W

) Using electrical analogy for the air film

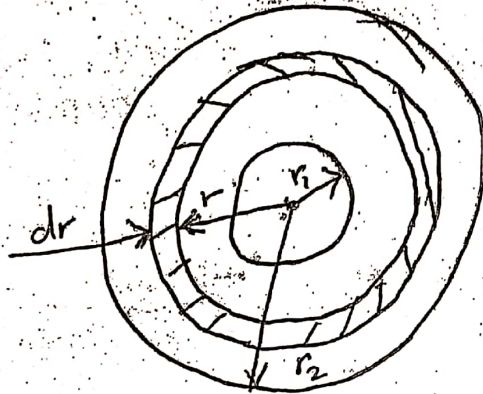
$$\dot{Q} = 116 = \frac{T - 15}{0.0675}, \quad \text{where } T \text{ is the temperature of outside surface}$$

$$T = (116 \times 0.0675) + 15 = 22.8^\circ\text{C}$$

i.e. Temperature of outside surface = 22.8°C

## The Sphere

Consider a hollow sphere of external radius  $r_2$  and internal radius  $r_1$  as shown in the figure below.



Let the inside and outside surface temperatures be  $T_1$  and  $T_2$ , and the thermal conductivity be  $k$ . Consider a small element of thickness  $dr$  at any radius  $r$ . The area of a ~~sphere is~~ the spherical element at radius  $r$  is  $4\pi r^2$ . Then using equation (1)

$$\dot{Q} = -kA \frac{dT}{dr} = -k4\pi r^2 \frac{dT}{dr}$$

Integrating,

$$\dot{Q} \int_{r_1}^{r_2} \frac{dr}{r^2} = -4\pi k \int_{T_1}^{T_2} dT$$

therefore

$$-\dot{Q} \left( \frac{1}{r_2} - \frac{1}{r_1} \right) = -4\pi k (T_2 - T_1)$$



$$\frac{\dot{Q} (r_2 - r_1)}{r_1 r_2} = 4\pi k (T_1 - T_2)$$

$$\text{i.e. } \dot{Q} = \frac{4\pi k r_1 r_2 (T_1 - T_2)}{(r_2 - r_1)} \quad \dots \dots \dots \text{(a)}$$

hence applying electrical analogy,  $(I = V/R)$ ,

$$R = \frac{r_2 - r_1}{4\pi k r_1 r_2} \quad \dots \dots \dots \text{(14)}$$

Introducing a mean Area  $A_m$ , then from equation (2)

$$\dot{Q} = \frac{k A_m (T_1 - T_2)}{x} = \frac{k A_m (T_1 - T_2)}{(r_2 - r_1)} \quad \dots \dots \dots \text{(b)}$$

Comparing equations (a) and (b)

$$A_m = 4\pi r_1 r_2$$

and a mean radius,  $r_m$ , can be defined

$$A_m = 4\pi r_1 r_2 = 4\pi r_m^2$$

Therefore,

$$r_m = \sqrt{r_1 r_2}$$

### Example 5

A small hemispherical oven is built of an inner layer of insulating firebrick 125 mm thick, and an outer covering of 85% magnesia 40 mm thick. The inner surface of the oven is at  $800^\circ\text{C}$  and the heat transfer coefficient for the outside surface

is  $10 \text{ W/m}^2 \text{ K}$ ; the room temperature is  $20^\circ \text{C}$ . Calculate the rate of heat loss through the hemisphere if the inside radius is  $0.6 \text{ m}$ . Take the thermal conductivities of fire brick and 85% magnesia as  $0.31$  and  $0.05 \text{ W/m K}$  respectively.

### Solution

From equation (14), for a hemisphere

$$\begin{aligned} \text{Resistance of fire brick} &= \frac{0.125}{2\pi \times 0.31 \times 0.6 \times 0.725} \\ &= 0.1478 \text{ K/W} \end{aligned}$$

$$\begin{aligned} \text{Resistance of 85\% magnesia} &= \frac{0.04}{2\pi \times 0.05 \times 0.725 \times 0.6} \\ &= 0.2295 \text{ K/W} \end{aligned}$$

For the outside surface from equation (8)

$$\begin{aligned} \text{Resistance of outside air film} &= \frac{1}{hA} \\ &= \frac{1}{10 \times 2\pi \times 0.765^2} \\ &= 0.0272 \text{ K/W} \end{aligned}$$

Hence,

$$\begin{aligned} \text{Total resistance } R_T &= 0.1478 + 0.2295 + 0.0272 \\ &= 0.4045 \text{ K/W} \end{aligned}$$

Then using equation (10)

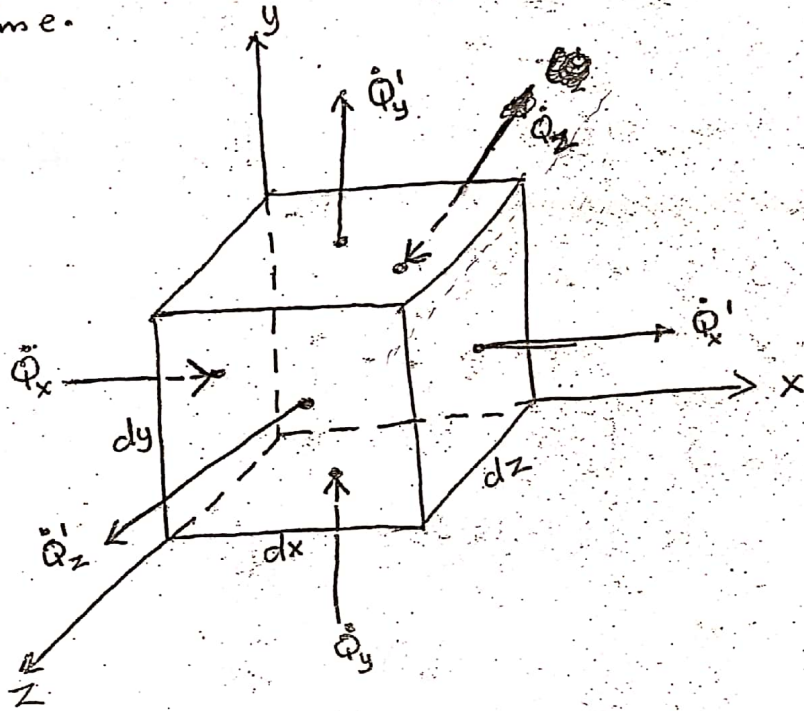
$$\dot{Q} = \frac{T_A - T_B}{R_T} = \frac{800 - 20}{0.4045} = 1930 \text{ W}$$

$\therefore$  Rate of heat loss from the oven =  $1.93 \text{ kW}$



## General Conduction Equation

A general conduction equation may be derived for a three-dimensional solid in which there is uniform internal heat generation and change of temperature with time.



Consider an element at the temperature,  $T$ , at any instant of time,  $t$ , within a homogeneous solid as shown in the figure above. Let the internal heat generation per unit volume be  $\dot{q}_g$ . Let the density of the material be  $\rho$ , the specific heat  $c$ , and the thermal conductivity  $k$ ; assume that these properties are uniform and constant with time.

By Fourier's law,

$$\dot{Q}_x = -k(dy dz) \frac{\partial T}{\partial x}$$

$$\dot{Q}_y = -k(dx dz) \frac{\partial T}{\partial y}$$

$$\dot{Q}_z = -k(dx dy) \frac{\partial T}{\partial z}$$

For x-direction

$$\dot{Q}'_x - \dot{Q}_x = \frac{\partial \dot{Q}_x}{\partial x} dx = -k \frac{\partial^2 T}{\partial x^2} dx dy dz$$

Similarly for the y and z directions.

Also Rate of heat generation =  $\dot{q}_g (dx dy dz)$

and

Rate of increase of energy of the element

= mass  $\times$  specific heat  $\times$  rate of change of temperature

$$= \rho (dx dy dz) c \frac{\partial T}{\partial t}$$

Hence an energy balance on the element gives

$$\dot{q}_g (dx dy dz) - [(\dot{Q}'_x - \dot{Q}_x) + (\dot{Q}'_y - \dot{Q}_y) + (\dot{Q}'_z - \dot{Q}_z)] = \rho c (dx dy dz) \frac{\partial T}{\partial t}$$

therefore,

$$\dot{q}_g - \left( -k \frac{\partial^2 T}{\partial x^2} - k \frac{\partial^2 T}{\partial y^2} - k \frac{\partial^2 T}{\partial z^2} \right) = \rho c \frac{\partial T}{\partial t}$$

Dividing through by k and introducing the thermal

diffusivity,  $\alpha = k/\rho c$ ,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}_g}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \text{--- (15)}$$

Equations using cylindrical or spherical coordinates may be derived in a similar way.

For one dimensional problems (e.g. infinite slab, infinitely long cylinder or a sphere), it is simpler to derive the equations directly.

(a) Infinite slab from equation (15)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}_g}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \text{--- (16)}$$

If no heat generation  $\dot{q}_g = 0$

For Steady state  $\frac{\partial T}{\partial x^2} + \frac{\dot{q}_g}{k} = 0$



b) Infinitely long cylinder (see figure on cylinder previously).  
 Apply an energy balance to an element of thickness,  $dr$ ,  
 and a unit length of cylinder

$$\dot{q}_g \cdot 2\pi r dr - \frac{\partial \dot{Q}}{\partial r} dr = \rho c \cdot 2\pi r dr \frac{\partial T}{\partial t} \quad \begin{matrix} L=1 \\ 2\pi r dr \times L \\ \text{Volume} \\ \text{of cylinder} \end{matrix}$$

therefore,

$$\dot{q}_g \cdot 2\pi r dr - \frac{\partial}{\partial r} \left( -k \cdot 2\pi r \frac{\partial T}{\partial r} \right) dr = \rho c \cdot 2\pi r dr \frac{\partial T}{\partial t}$$

therefore

$$\dot{q}_g r - \left( kr \frac{\partial^2 T}{\partial r^2} + k \frac{\partial T}{\partial r} \right) = \rho c r \frac{\partial T}{\partial t}$$

$$\text{i.e.} \quad \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\dot{q}_g}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \dots \dots \dots (17)$$

(c) Sphere (see previous figure on sphere)

~~$$\dot{q}_g \cdot 4\pi r^2 dr - \frac{\partial}{\partial r} \left( -k \cdot 4\pi r^2 \frac{\partial T}{\partial r} \right) dr = \rho c \cdot 4\pi r^2 dr \frac{\partial T}{\partial t}$$~~

$$\dot{q}_g \cdot 4\pi r^2 dr - \frac{\partial \dot{Q}}{\partial r} dr = \rho c \cdot 4\pi r^2 dr \frac{\partial T}{\partial t}$$

therefore

$$\dot{q}_g \cdot 4\pi r^2 dr - \frac{\partial}{\partial r} \left( -k \cdot 4\pi r^2 \frac{\partial T}{\partial r} \right) dr = \rho c \cdot 4\pi r^2 dr \frac{\partial T}{\partial t}$$

$$\text{i.e.} \quad \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} + \frac{\dot{q}_g}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \dots \dots \dots (18)$$

For steady state cases the right hand side of equations (16), (17) and (18) becomes zero, and the equations become ordinary differential equations.

### Example 6

A hollow cylindrical copper conductor of 30 mm outside diameter and 14 mm inside diameter has a current density of  $40 \text{ A/mm}^2$ . The external surface is ~~covered~~ covered with a uniform layer of insulation of thickness 10 mm, and the ambient temperature is  $10^\circ\text{C}$ . Neglecting axial conduction and assuming that the temperature of the insulation must not exceed  $135^\circ\text{C}$  at any point, calculate:

- (i) the heat required to be removed per unit time by forced cooling from the inside of the conductor;
- (ii) the temperature at the inside surface of the conductor.

### Data

Thermal conductivity of copper =  $380 \text{ W/mK}$ ; thermal conductivity of insulating material =  $0.3 \text{ W/mK}$ ; heat transfer coefficient at outside surface =  $40 \text{ W/m}^2\text{K}$ ; electrical resistivity of copper =  $2 \times 10^{-5} \text{ } \Omega \text{ mm}$ .

### Solution

From equation (17), for steady state

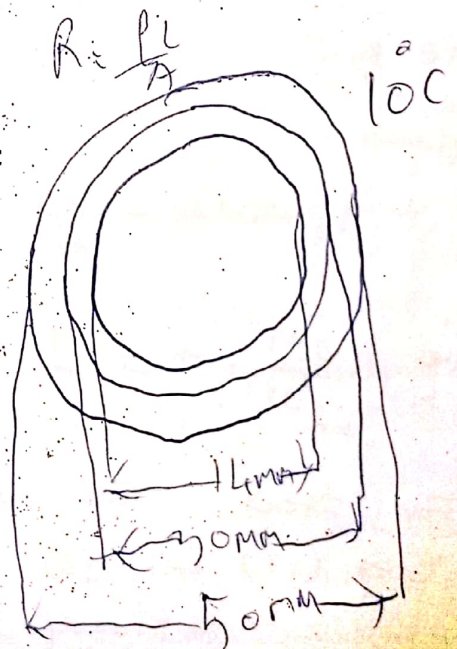
$$\frac{d^2T}{dr^2} + \frac{1}{r} \frac{dT}{dr} + \frac{q_g}{k} = 0$$

or

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) = - \frac{q_g}{k}$$

Integrating,

$$r \frac{dT}{dr} = - \frac{q_g r^2}{2k} + C_1$$





herefore,

$$\frac{dT}{dr} = -\frac{\dot{q}_g r}{2k} + \frac{C_1}{r} \dots \dots \dots (a)$$

Integrating,

$$T = -\frac{\dot{q}_g r^2}{4k} + C_1 \log_e r + C_2 \dots \dots \dots (b)$$

where  $C_1$  and  $C_2$  are constants.

The heat generated per unit volume due to the current flowing is given by

$$\dot{q}_g = \frac{I^2 R}{AL}$$

where  $I$  is the current,  $R$  the electrical resistance,  $A$  the cross-sectional area and  $L$  the length.

Current density,  $J = I/A$  and resistance,  $R = sL/A$

where  $s$  is the electrical resistivity of the conductor material.

$$\text{i.e. } \dot{q}_g = \frac{J^2 A^2 s L}{AL A} = J^2 s \dots \dots \dots (19)$$

$$= 40^2 \times 2 \times 10^{-5} \text{ W/mm}^3 = 32 \times 10^6 \text{ W/m}^3$$

(=135°C)

(v) The maximum temperature of the insulation occurs at the interface between the insulation and the copper tube. Hence for the insulation, using equation (12)

$$\text{Heat transfer to the outside, } \dot{Q}_o = \frac{2\pi \times 0.3 (135 - T)}{\log_e (25/15)}$$

where  $T$  is the temperature of the outside surface of the insulation,

$$\text{i.e. } 135 - T = 0.271 \dot{Q}_o \dots \dots \dots (c)$$

For the heat transferred from the outside surface of the insulation by convection, from equation (3)

$$\dot{Q}_o = hA(T - T_{\text{fluid}})$$

$$\text{i.e. } \dot{Q}_o = 40 \times 2\pi \times 0.025 (T - 10)$$

Therefore,

$$T - 10 = 0.159 \dot{Q}_o \quad \dots \dots \dots (d)$$

Adding equations (c) and (d)

$$135 - 10 = 0.43 \dot{Q}_o \Rightarrow \dot{Q}_o = 290.7 \text{ W}$$

Total heat generated internally =  $\dot{q}_g \times \text{volume}$

$$= 32 \times 10^6 \times \frac{\pi}{4} (0.03^2 - 0.014^2)$$

$$= 17693.5 \text{ W}$$

Hence,

$$\begin{aligned} \text{Heat removed from inside of conductor} &= (17693.5 - 290.7) \text{ W} \\ &= 17.4 \text{ kW} \end{aligned}$$

(i) Two boundary conditions are required to find constants  $C_1$  and  $C_2$  and hence to obtain the solution to equation (b)

At the inside surface of the conductor

$$\text{Heat supplied to conductor} = -kA \left( \frac{dT}{dr} \right)_{r=0.007} = -17400 \text{ W}$$

Therefore

$$\left( \frac{dT}{dr} \right)_{r=0.007} = \frac{17400}{380 \times \pi \times 0.014} = 1041.1 \text{ K/m}$$

Substituting in equation (a)

$$1041.1 = - \left[ \frac{32 \times 10^6 \times 0.007}{2 \times 380} \right] + \frac{C_1}{0.007}$$

$$\text{Therefore, } C_1 = 9.351$$



At the outside surface of the conductor,  $T = 135^\circ\text{C}$ ,  
hence in equation (b)

$$135 = - \left[ \frac{32 \times 10^6 \times 0.015^2}{4 \times 380} \right] + 9.351 \log_e(0.015) + C_2$$

therefore,

$$C_2 = 179$$

Therefore the complete solution for the temperature distribution in the conductor is

$$T = - \left( \frac{32 \times 10^6}{4 \times 380} \right) r^2 + 9.351 \log_e r + 179$$

hence, at the inside surface,  $r = 0.007 \text{ m}$

$$\begin{aligned} T &= - \left[ \frac{32 \times 10^6}{4 \times 380} \right] + 9.351 \log_e 0.007 + 179 \\ &= 131.6^\circ\text{C} \end{aligned}$$

## Forced Convection

The study of forced convection is concerned with the transfer of heat between a moving fluid and a solid surface. To apply Newton's law of cooling (equation (3)), it is necessary the heat transfer coefficient,  $h$ , has to be known.  $h$  is given by  $k/\delta$ , where  $k$  is the thermal conductivity of the fluid and  $\delta$  is the thickness of the fluid film on the surface. The problem becomes that of finding out a value for  $\delta$  in terms of the fluid properties and the velocity;  $\delta$  depends on the type of fluid flow across the surface, which is governed by the Reynolds number.

The Reynolds number is a dimensionless group given by

$$Re = \frac{\rho C L}{\mu} \quad \text{or} \quad \frac{C L}{\nu}$$

where  $\rho$  is the fluid density,  $C$  the fluid velocity,  $L$  the characteristic linear dimension,  $\mu$  the dynamic viscosity of the fluid and  $\nu$  the kinematic viscosity of the fluid given by  $\mu/\rho$ .

Many results in heat transfer are derived from experiment and for many problems no mathematic solution exist and empirical values are essential. These empirical values can be generalised using dimensional analysis.



## Dimensional Analysis

In order to apply dimensional analysis it is necessary to know all the variables upon which the desired function depends. The results must apply to geometrically similar bodies, therefore one of the variables must always be a characteristic linear dimension.

Consider the dimensional analysis for forced convection, assuming that the effects of free convection, due to density differences are negligible. It is found that the heat transfer coefficient,  $h$ , depends on the fluid viscosity,  $\mu$ , the fluid density,  $\rho$ , the thermal conductivity of the fluid,  $k$ , the specific heat capacity of the fluid,  $c$ , the temperature difference between the surface and the fluid,  $\Delta T$ , and the fluid velocity,  $C$ . Therefore,

$$h = f(\mu, \rho, k, c, \Delta T, C, L) \quad \dots \dots \dots (20)$$

where  $L$  is a characteristic linear dimension and  $f$  is some function.

Equation (20) can be rewritten as follows:

$$h = A \mu^{a_1} \rho^{b_1} k^{c_1} c^{d_1} \Delta T^{e_1} C^{f_1} L^{g_1} + B \mu^{a_2} \rho^{b_2} k^{c_2} c^{d_2} \Delta T^{e_2} C^{f_2} L^{g_2} + \text{etc.} \quad (21)$$

where  $A$  and  $B$  are constants and  $a_1, b_1, c_1, d_1, e_1, f_1, g_1$  are arbitrary indices. Each term on the right hand side of equation (21) must have the same dimensions as dimensions of  $h$ . Considering the first term only,

$$\text{Dimensions of } h = \text{Dimensions of } (\mu^{a_1} \rho^{b_1} k^{c_1} \Delta T^{e_1} C^{f_1} L^{g_1})$$

Each of the properties in the equation can be expressed in terms of five fundamental dimensions; ~~of~~ these are mass,  $M$ , length,  $L$ , time,  $T$ , temperature,  $t$ , and heat,  $H$ .

For  $h$  the units are  $\frac{W}{m^2 K}$ , i.e.  $\frac{H}{L^2 T t}$

For  $\mu$  the units are  $\frac{kg}{ms}$ , i.e.  $\frac{M}{L T}$

For  $k$  the units are  $\frac{W}{m K}$ , i.e.  $\frac{H}{L T t}$

For  $\rho$  the units are  $\frac{kg}{m^3}$ , i.e.  $\frac{M}{L^3}$

For  $c$  the units are  $\frac{kJ}{kg K}$ , i.e.  $\frac{H}{M t}$

For  $\Delta T$  the units are  $K$ , i.e.  $t$

For  $C$  the units are  $\frac{m}{s}$ , i.e.  $\frac{L}{T}$

For  $l$  the units are  $m$ , i.e.  $L$

Hence substituting

$$\frac{H}{L^2 T t} = \left(\frac{M}{L T}\right)^a \times \left(\frac{M}{L^3}\right)^b \times \left(\frac{H}{L T t}\right)^c \times \left(\frac{H}{M t}\right)^d \times (t)^e \times \left(\frac{L}{T}\right)^f \times (L)^g$$

$$\text{i.e. } \frac{H}{L^2 T t} = (M)^{a+b-d} \times (L)^{f+g-a-3b-c} \times (T)^{-a-c-f} \times (t)^{e-c-d} \times (H)^{c+d}$$

For the dimensions of each side of the equation to be the same the power to which each fundamental dimension is raised must be the same on both sides of the equation. Therefore, equating indices,

For  $H$ :  $1 = c + d$

For  $L$ :  $-2 = f + g - 3a - 3b - c$

For  $T$ :  $-1 = -a - c - f$

For  $t$ :  $-1 = e - c - d$

∴  $M$ :  $a = a + b - d$



There are five equations and seven unknowns, therefore a solution can only be obtained in terms of two of the indices. Expressing a, b, c, e and g in terms of d and f, it can be shown that

$$a = d - f; \quad b = f; \quad c = 1 - d; \quad e = 0; \quad g = f - 1$$

Substituting these values in equation (21)

$$A \mu^{(d_1 - f_1)} \rho^{f_1} k^{(1 - d_1)} c^{d_1} \Delta T^0 L^{f_1 (f_1 - 1)} + B \mu^{(d_2 - f_2)} \rho^{f_2} k^{(1 - d_2)} c^{d_2} \Delta T^0 L^{f_2 (f_2 - 1)} + \dots$$

$$h = A \frac{k}{L} \left(\frac{c\mu}{k}\right)^{d_1} \left(\frac{\rho c L}{\mu}\right)^{f_1} + B \frac{k}{L} \left(\frac{c\mu}{k}\right)^{d_2} \left(\frac{\rho c L}{\mu}\right)^{f_2} + \dots$$

Hence,

$$\frac{hL}{k} = KF \left[ \left(\frac{c\mu}{k}\right), \left(\frac{\rho c L}{\mu}\right) \right]$$

where K is a constant and F is the Reynolds number, Re. The dimensionless group,  $hL/k$ , is called the Nusselt number,  $Nu$ ; the dimensionless group,  $c\mu/k$ , is called the Prandtl number,  $Pr$ ; and the dimensionless group,  $\rho c L/\mu$ , is called the Reynolds number,  $Re$ .

$$Nu = KF(Pr, Re) \quad \dots \dots \dots (22)$$

Experiments can be performed to determine K and the nature of the function F.

When evaluating  $Nu$ ,  $Pr$  and  $Re$  it is necessary to take the properties at a suitable mean temperature, since the properties vary with temperature. For cases where the temperature of the bulk of the fluid is not very different from the temperature of the solid surface, then fluid properties are evaluated at the mean bulk temperature of the fluid. When the temperature difference is large, errors may be caused by using

a mean bulk temperature, and a film mean film temperature is sometimes used, defined by

$$T_f = \frac{T_b + T_w}{2}$$

where  $T_b$  is the mean bulk temperature and  $T_w$  the surface temperature.

When using an empirical equation it is important to know at what reference temperature the properties were evaluated by the experimenter. It should be noted that Prandtl number,  $Pr = c_p \mu / k$ , consists entirely of fluid properties and therefore is itself a property.

~~Some~~

### Example 7

Calculate the heat transfer coefficient for water flowing through a 25 mm diameter tube at the rate of 1.5 kg/s when the ~~bulk~~ mean bulk temperature is  $40^\circ\text{C}$ . For turbulent flow of a liquid,

$$Nu = 0.0243 Re^{0.8} \times Pr^{0.4}$$

where the characteristic dimension of length is the tube diameter and all the properties are evaluated at the mean bulk temperature.

### Solution

First it is necessary to ascertain whether the flow is turbulent or laminar. For flow through a tube it can be assumed that the flow is turbulent where  $Re > 2100$  approximately. The properties of water can be taken from steam tables.



$$\text{Volume flow} = 1.5 \times V_f = 1.5 \times 0.001 = 0.0015 \text{ m}^3/\text{s}$$

$$\text{Velocity in tube, } C = \frac{0.0015 \times 4}{\pi \times 0.025^2} = 3.06 \text{ m/s}$$

$$Re = \frac{\rho C D}{\mu} = \frac{C D}{\nu} = \frac{3.06 \times 0.025}{0.001 \times 651 \times 10^{-6}} = 117500$$

Therefore the flow is turbulent

From tables,  $Pr = 4.3$ , hence,

$$Nu = 0.0243 \times (117500)^{0.8} \times (4.3)^{0.4}$$

$$= 0.0243 \times 11377 \times 1.792 = 495.5$$

$$\text{i.e. } Nu = \frac{hD}{k} = 495.5$$

Therefore,

$$h = \frac{495.5 \times 632 \times 10^{-6}}{0.025} = 12.53 \text{ kW/m}^2\text{K}$$

i.e. Heat transfer coefficient =  $12.53 \text{ kW/m}^2\text{K}$

### Reynolds Analogy

Reynolds postulated that the heat transfer from a solid surface or wall is similar to the ~~momentum~~ transfer of fluid momentum from the surface, and hence that it is possible to express the heat transfer in terms of the frictional resistance to the flow.

Consider turbulent flow. It can be assumed that particles of mass,  $m$ , transport heat and momentum to and from the wall, moving perpendicular to the surface. Then on



average

Heat transferred per unit area,  $\dot{q} = \dot{m} c \Delta T$ .

where  $c$  is the specific heat capacity of the fluid and  $\Delta T$  the temperature difference between the surface and the bulk of the fluid. Also, the rate of change of momentum across the stream is given by

$$\dot{m}(C - C_w) = \dot{m} C$$

where  $C$  is the velocity of the bulk of the fluid and  $C_w$  the fluid velocity at the surface = 0. Then

$$\text{Force per unit area} = \tau_w = \dot{m} C$$

where  $\tau_w$  is the shear stress in the fluid at the wall.

Combining the equations for heat flow and momentum transfer, then

$$\frac{\dot{q}}{c \Delta T} = \frac{\tau_w}{C}$$

$$\text{or } \dot{q} = \frac{\tau_w c \Delta T}{C} \quad \dots \dots \dots (23)$$

For turbulent flow in practice there is always a thin layer of fluid on the surface in which viscous effects predominate. This film is known as the laminar sub layer. In this layer heat transfer is purely by conduction.

Therefore, from Fourier's law, for unit area

$$\dot{q} = -k \left( \frac{dT}{dy} \right)_{y=0}$$

where  $k$  is the thermal conductivity of the fluid and  $y$  the distance from the surface perpendicular to the surface.

Also, for viscous flow

$$\text{Shear stress, } \tau = \mu \frac{dc}{dy}$$



Hence, at the wall

$$\tau_w = \mu \left( \frac{dc}{dy} \right)_{y=0}$$

where  $\mu$  is the fluid viscosity and  $C$  the fluid velocity.

Since the laminar sublayer is very thin it may be assumed that the temperature and velocity vary linearly with the distance from the wall,  $y$ ,

i.e.  $\dot{q} = -\frac{k\Delta T}{\delta_b}$  and  $\tau_w = \frac{\mu C}{\delta_b}$

where  $\delta_b$  is the thickness of the laminar sub-layer.

Eliminating  $\delta_b$ , and neglecting the minus sign

$$\frac{\dot{q}}{k\Delta T} = \frac{\tau_w}{\mu C}$$

It is seen that this equation is ~~similar~~ identical to equation (23) when  $C = \frac{k}{\mu}$

i.e. when

$$\frac{c\mu}{k} = 1 \text{ or } Pr = 1$$

Therefore for fluids whose Prandtl number is approximately unity the simple Reynolds analogy can be applied.

For a unit surface area,  $\dot{q} = h\Delta T$ , therefore substituting in equation (23)

$$\frac{h}{c} = \frac{\tau_w}{c}$$

Dividing through by  $\rho C$ , where  $\rho$  is the mean density of the fluid,

$$\frac{h}{\rho C} = \frac{\tau_w}{\rho C^2}$$

Both sides of this equation are dimensionless. The term on the left-hand side is called the Stanton number,  $St$ ,

$$\text{i.e. } St = \frac{h}{\rho c_e} \quad (24)$$

A dimensionless friction factor,  $f$ , is defined by

$$f = \frac{\tau_w}{\rho C^2 / 2} \quad (25)$$

Therefore for Reynolds analogy

$$St = \frac{f}{2} \quad (26)$$

The Stanton number can be re-written as

$$St = \frac{h}{\rho c_e} = \frac{h k}{k} \times \frac{\mu}{\rho C L} \times \frac{k}{c \mu} = \frac{Nu}{Re Pr}$$

$$\text{i.e. } St = \frac{Nu}{Re Pr} \quad (27)$$

For flow in a pipe of diameter,  $D$ , the resistance to flow over a unit length is given by

$$\text{Resistance} = \tau_w \pi D = \frac{\Delta P \pi D^2}{4}$$

where  $\Delta P$  is the pressure drop in unit length

$$\text{i.e. } \tau_w = \frac{\Delta P D}{4} \quad (28)$$

An important factor in pumping system design is the pumping power required. The pumping power is the rate at which work is done in overcoming the frictional resistance, i.e. for flow in a pipe.



(10)

pumping power per unit length,  $\dot{W} = \tau_w \pi D C$

Also from equation (23)

Heat flow per unit area,  $\dot{q} = \frac{\tau_w c \Delta T}{C}$

i.e. Heat flow per unit length =  $\frac{\tau_w c \Delta T \pi D}{C}$

Then the ratio of the pumping power,  $\dot{W}$ , to the rate of heat flow,  $\dot{Q}$ , can be expressed as

$\frac{\dot{W}}{\dot{Q}} = \frac{\tau_w \pi D C C}{\tau_w c \Delta T \pi D} = \frac{C^2}{c \Delta T}$  (29)

For a heat exchanger,  $\Delta T$  is the log mean temperature difference,  $\Delta T_{loge}$ .

It is seen from equation (29) that the power required for a given heat transfer rate can be reduced by decreasing the velocity of flow,  $C$ . However, a reduction in fluid velocity means that the required surface area must be increased.

Example 2

Air is heated by passing it through a 25 mm bore copper tube which is maintained at 280°C. The air enters at 15°C and leaves at 270°C at a mean velocity of 30 m/s. Using Reynolds analogy, calculate the length of the tube and the pumping power required. For turbulent flow in a tube take  $f = 0.0791 (Re)^{-1/4}$ , and all properties at mean film temperature. Take the mean temperature difference as  $\Delta T_{loge} = (\Delta T_1 - \Delta T_2) / \log_e (\Delta T_1 / \Delta T_2)$ .

## Solution

The mean film temperature can be found from ~~equation~~

$$T_f = \frac{T_b + T_w}{2} = \frac{1}{2} \left( \frac{15 + 270}{2} + 280 \right) = \frac{142.5 + 280}{2} = 211.25^\circ\text{C}$$

From tables at  $T_f = 211.25^\circ\text{C} = 484.25\text{K}$  the properties of air can be found:

$$Re = \frac{CD}{\gamma} = \frac{30 \times 0.025}{3.591 \times 10^{-5}} = 20900$$

Therefore,

$$f = \frac{0.0791}{(20900)^{1/4}} = \frac{0.0791}{12.01} = 0.00658$$

From equations (26) and (27)

$$St = \frac{Nu}{Re Pr} = \frac{f}{2} = \frac{0.00658}{2} = 0.00329$$

$$\text{i.e. } Nu = 0.00329 \times 20900 \times 0.681 = 46.8$$

Therefore,

$$\frac{hD}{k} = 46.8$$

$$h = \frac{46.8 \times 3.938 \times 10^{-5}}{0.025} = 0.0737 \text{ kW/m}^2\text{K}$$

$$\text{Mass flow of air} = \frac{\pi}{4} \times 0.025^2 \times 30 \times 0.73 = 0.01075 \text{ kg/s}$$

Hence,

$$\text{Heat received by air} = mc(T_{a2} - T_{a1})$$

$$= 0.01075 \times 1.027 \times (270 - 15)$$

$$= 2.815 \text{ kW}$$

Also

$$\dot{Q} = hA\Delta T = 2.815$$

and

$$\Delta T_{\log e} = \frac{\Delta T_1 - \Delta T_2}{\log_e(\Delta T_1/\Delta T_2)} = \frac{(280 - 15) - (280 - 270)}{\log_e[(280 - 15)/(280 - 270)]} = 77.9 \text{ K}$$



$$\dot{Q} = 2.815 = 0.0737 \times 77.9 \times A$$

Therefore,  $A = \frac{2.815}{0.0737 \times 77.9} = 0.49 \text{ m}^2$

tube length,  $L = \frac{A}{\pi D} = \frac{0.49}{\pi \times 0.025} = 6.24 \text{ m}$

From equation (29)

$$\frac{\dot{W}}{\dot{Q}} = \frac{C^2}{CA^2}$$

Therefore,

$$\dot{W} = \frac{2.815 \times 30^2}{1.027 \times 77.9} = 31.7 \text{ W}$$

i.e. Pumping power = 31.7 W

Example 9

In a 25 mm diameter tube the pressure drop per metre length is 0.0002 bar at a section where the mean velocity is 24 m/s, and the mean specific heat capacity of the gas is 1.13 kJ/kg.K. Calculate the heat transfer coefficient.

Solution

For 1 metre length of tube  $\Delta P = 0.0002 \text{ bar}$

From equation (28)

$$\tau_w = \frac{\Delta P D}{4} = \frac{10^5 \times 0.0002 \times 25}{4 \times 10^3} = 0.125 \text{ N/m}^2$$

From equation (25)

$$f = \frac{\tau_w}{\rho C^2 / 2} = \frac{2 \times 0.125}{\rho C^2}$$

Also from equation (2.6)

$$St = \frac{f}{2} = \frac{2 \times 0.125}{2 \rho C^2}$$

and equation (2.4)

$$St = \frac{h}{\rho C} = \frac{2 \times 0.125}{2 \rho C^2}$$

$$\therefore h = \frac{0.125 \rho C}{\rho C^2} = \frac{0.125 \times 1.13}{24}$$

$$= 0.00588 \text{ kW/m}^2 \text{ K}$$

$$\text{Heat transfer coefficient} = 5.88 \text{ W/m}^2 \text{ K}$$

## Natural Convection

Heat transfer by natural or free convection is due to differences in density in the fluid causing a natural circulation and hence a transfer of heat. When there is no forced convection then heat is transferred entirely by natural convection (when radiation is negligible). The heat transfer in this case depends on the coefficient of cubic expansion,  $\beta$ , which is given by

$$\beta = \frac{\rho_1 - \rho_2}{\rho_2 \Delta T}$$

where  $\Delta T$  is the temperature difference between two parts of the fluid of density  $\rho_1$  and  $\rho_2$ .

The upthrust per unit volume of fluid is  $(\rho_1 - \rho_2)g$ , and the velocity of the convection current is dependent on the upthrust, i.e. natural convection depends on

$$(\rho_1 - \rho_2)g = \rho_2 \beta \Delta T g$$



The heat transfer depends on the fluid viscosity, the thermal conductivity of the fluid, and a characteristic dimension of length. Since the coefficient of cubic expansion  $\beta$ , and the local acceleration due to gravity,  $g$ , do not have a separate effect on the heat transfer, then only the product,  $\beta g$ , is considered. For a dimensional analysis for natural convection:

$$h = A \mu^{a_1} \rho^{b_1} k^{c_1} \Delta T^{e_1} (\beta g)^{f_1} L^{g_1} + B \mu^{a_2} \rho^{b_2} k^{c_2} \Delta T^{e_2} (\beta g)^{f_2} L^{g_2} + \text{etc.}$$

Then from the same procedure as for forced convection it can be shown that

$$Nu = KF \left[ \left( \frac{c\mu}{k} \right), \left( \frac{\beta g \rho^2 L^3 \Delta T}{\mu^2} \right) \right]$$

or  $Nu = KF (Pr, Gr)$

where  $Gr = \frac{\beta g \rho^2 L^3 \Delta T}{\mu^2} = \frac{\beta g L^3 \Delta T}{\nu^2}$

$Gr$  is the Grashof number

In many cases of natural convection it is possible to use an approximate equation to evaluate the heat transfer coefficient,  $h$ . For example, for natural convection from a horizontal pipe

$$h = 1.32 \left( \frac{\Delta T}{D} \right)^{1/4} \quad \text{when } 10^5 < Gr < 10^9$$

and  $h = 1.25 \Delta T^{1/3} \quad \text{when } 10^9 < Gr < 10^{12}$

where  $h$  is in  $W/m^2K$ ,  $\Delta T$  is in  $K$ ,  $D$  is in  $m$ .



### Example 10

Calculate the rate of heat loss by natural convection per unit length from a horizontal pipe of diameter 150 mm, the surface of which is at  $277^\circ\text{C}$ . The room temperature is  $17^\circ\text{C}$ . It has been shown that for a horizontal cylinder

$$Nu = 0.53 (Gr Pr)^{1/4}$$

Where the properties are evaluated at the mean film temperature. Take coefficient of cubic expansion,  $\beta$ , as  $1/T$ , where  $T$  is the absolute temperature of air.

### Solution

From tables, at a mean film temperature of  $\frac{1}{2}(550 + 290) = 420\text{K}$ ,  $Pr = 0.686$

$$Gr = \frac{\beta g \Delta T D^3}{\nu^2} = \frac{9.81 \times (277 - 17) \times 0.15^3}{290 \times (2.822 \times 10^{-5})^2} = 37.27 \times 10^6$$

Substituting

$$Nu = 0.53 (37.27 \times 10^6 \times 0.686)^{1/4} = 37.7$$

$$\text{i.e. } Nu = \frac{hD}{k} = 37.7$$

$$h = \frac{37.7 \times 3.635 \times 10^{-5}}{0.15} = 0.00913 \text{ kW/m}^2\text{K}$$

$$\dot{Q} = hA(T_w - T) = 0.00913 \times \pi \times 0.15 \times 1 \times (277 - 17) = 1.119 \text{ kW}$$

i.e. Heat loss per metre length = 1.119 kW